

THE PROBABILITY OF FINDING A CARPARK

Suppose a tournament with $2n$ teams is played at a Petanque club on two terrains containing $n = x + y$ marked pistes. x is the number of pistes on the club's usual terrain and y is the number of less desirable pistes on a nearby carpark. The pistes for a round of the tournament with n games are allocated by a random draw.

What is the probability of a team being allocated a *carpark-piste*?

It turns out that this probability is

$$\Pr \{ \text{carpark-piste} \} = \frac{y}{n} \quad (1)$$

This is easily deduced from a classical definition of probability (Spiegel¹).

Suppose an event E can happen in h ways out of a total of n possible equally likely ways. Then the probability of occurrence of the event (called its *success*) is denoted by

$$p = \Pr \{ E \} = \frac{h}{n} \quad (2)$$

The probability of non-occurrence of the event (called its *failure*) is denoted by

$$q = \Pr \{ \text{not } E \} = \frac{n - h}{n} = 1 - \frac{h}{n} = 1 - p = 1 - \Pr \{ E \}. \quad (3)$$

Thus $p + q = 1$ or $\Pr \{ E \} + \Pr \{ \text{not } E \} = 1$.

Note that the probability of an event is a number between 0 and 1. If the event cannot occur, its probability is 0. If it must occur, i.e. its occurrence is *certain*, its probability is 1.

What is the probability of a team being allocated carpark-pistes in k rounds of the tournament?

It turns out that this probability is

$$\Pr \{ k \text{ carpark-pistes} \} = \left(\frac{y}{n} \right)^k \quad (4)$$

¹ Spiegel, M.R., 1972, *Theory and Problems of Statistics*, Schaum's Outline Series, McGraw-Hill Book Company, New York.

This can be deduced from the probability of occurrence of *independent events*², where in general if $E_1, E_2, E_3, \dots, E_k$ are independent events then

$$\Pr \{E_1 E_2 E_3 \dots E_k\} = \Pr \{E_1\} \Pr \{E_2\} \Pr \{E_3\} \dots \Pr \{E_k\} = p_1 p_2 p_3 \dots p_k \quad (5)$$

In our case we denote the event of being allocated a carpark-piste by E and

$\Pr \{E\} = \Pr \{\text{carpark-piste}\} = \frac{y}{n}$. Since the allocation of pistes in one round is

independent of the allocation of pistes in another round, we may write the probability of being allocated a carpark-piste in two rounds as

$$\Pr \{EE\} = \Pr \{2 \text{ carpark-pistes}\} = \left(\frac{y}{n}\right)\left(\frac{y}{n}\right) = \left(\frac{y}{n}\right)^2$$

and in three rounds as

$$\Pr \{EEE\} = \Pr \{3 \text{ carpark-pistes}\} = \left(\frac{y}{n}\right)\left(\frac{y}{n}\right)\left(\frac{y}{n}\right) = \left(\frac{y}{n}\right)^3$$

In k rounds we have equation (4)

There was one such Petanque tournament conducted recently where $2n = 38$ teams took part but the club's usual terrain contained only $x = 11$ marked pistes. So an additional $y = 8$ pistes were marked out on a nearby carpark. These pistes were quite rough (the local youth often use the carpark for speedway practice) and players were apprehensive at the draw for piste allocation. The following page shows the ranking of the 38 teams after four qualifying rounds of the tournament (using the Swiss System).

A single team was allocated a carpark-piste in **each** qualifying round. The probability of this occurrence is:

$$\Pr \{EEEE\} = \Pr \{4 \text{ carpark-pistes}\} = \left(\frac{y}{n}\right)^4 = \left(\frac{8}{19}\right)^4 = 0.0314$$

This rare event would occur approximately 3 times in 100 similar tournaments.

They were gracious in accepting their "good luck", even drawing the numbers (including their own) in the final random draw for pistes.

² If the occurrence or non-occurrence of E_1 does not affect the probability of occurrence of E_2 then E_1 is independent of E_2

Rankings after four Qualifying Rounds

1	2	3	4	Rank	Player 1	Player 2	Score	BHN	fBHN	Games	Points
x	x	x	x	1.	Dufresne, Patrick	Dufresne, Lynn	4	10	32	4:0	50:19
✓	x	✓	✓	2.	Lubin, Pierrot	Mercier, Huguette	4	9	40	4:0	52:26
✓	x	✓	✓	3.	Lousteau, Michel	Lubin, Josiane	4	6	39	4:0	52:23
✓	x	✓	✓	4.	Bahler, Guy	Bouchon, Virginie	3	12	38	3:1	38:34
x	✓	✓	✓	5.	Masson, Denis	Langlois, Juliette	3	10	36	3:1	46:32
✓	✓	x	x	6.	Mercier, Guy	Anthian, Christiane	3	9	36	3:1	42:28
✓	✓	x	✓	7.	Grancourt, Sylvio	Grancourt, Danielle	3	8	36	3:1	41:26
✓	✓	✓	✓	8.	Langlois, Stephane	Masson, Annick	3	8	34	3:1	43:14
x	x	x	✓	9.	Wilmann, Vivian	Lebrasse, Medgee	3	8	34	3:1	44:34
✓	✓	x	✓	10.	Vencatasamy, Frederick	Deramond, Adeline	3	7	37	3:1	49:26
✓	✓	✓	x	11.	Ramond, Guillaume	Marie, Eileen	3	7	33	3:1	44:21
x	✓	x	x	12.	Florent, Cyril	Florent, Chantal	3	7	30	3:1	48:27
✓	x	x	✓	13.	Leconte, Eric	Mangan, Kate	2	11	34	2:2	37:38
x	x	✓	✓	14.	Vissenjoux, Louis	Barter, Terry	2	10	29	2:2	36:43
x	✓	x	✓	15.	Canal, Jean Claude	Care, Alice	2	9	35	2:2	39:47
✓	x	x	x	16.	Bommarito, Bernard	Bommarito, Danielle	2	9	33	2:2	46:34
x	x	✓	x	17.	Bernhard, Luc	Papotto, Elisa	2	9	31	2:2	35:32
x	✓	x	✓	18.	Masson, Eddy	Langlois, Muriel	2	8	28	2:2	36:34
✓	✓	✓	✓	19.	Hamon, Patrick	Deramond, Lisa	2	8	25	2:2	38:36
✓	✓	✓	✓	20.	Ally, Christian	Coulon, Monique	2	7	37	2:2	32:36
x	x	✓	✓	21.	Bradburn, Clare	Bradburn, Brian	2	7	37	2:2	34:41
✓	x	x	✓	22.	Peachey, Lynn	Peachey, Bill	2	7	32	2:2	26:41
x	✓	x	x	23.	Neilsen, Sue	Neilsen, Trevor	2	6	35	2:2	37:42
✓	x	✓	x	24.	Lablache, Tony	Boissezon, Cecile	2	5	34	2:2	42:43
x	✓	✓	✓	25.	Mayor, Santiago	Mayor, Pierette	2	5	32	2:2	41:41
✓	✓	x	x	26.	Parley, Jean	Parley, Charles	1	11	28	1:3	35:47
✓	x	x	✓	27.	Cure, Desiree	Savanah, Tino	1	10	29	1:3	21:46
✓	x	✓	✓	28.	Oudin, Alain	Lasplaces, Suzelle	1	10	27	1:3	35:36
x	✓	x	✓	29.	Frederic, Jean	Frederic, Gyliane	1	8	33	1:3	28:46
x	✓	✓	x	30.	Lucette, Christian	Darbinian, Sasha	1	8	31	1:3	34:41
x	✓	✓	✓	31.	Finette, Gerard	Darbinian, Anastasea	1	8	28	1:3	15:41
✓	✓	x	x	32.	Sewhee, Kenny	Mini Kuan, Lim	1	8	27	1:3	32:50
✓	✓	✓	x	33.	Wales, Gloria	Lockwood, Bill	1	8	25	1:3	26:51
✓	x	✓	✓	34.	Blackburn, Percy	Paruit, Lisette	1	7	30	1:3	33:46
x	x	✓	x	35.	Leconte, Martial	Leconte, Claudie	1	6	30	1:3	35:48
✓	✓	✓	x	36.	McIntyre, Graeme	McIntyre, Laraine	1	6	24	1:3	32:48
✓	✓	✓	x	37.	Paruit, Neville	Canal, Maria Ines	0	7	27	0:4	33:50
x	✓	✓	x	38.	Forrest, Jim	Forrest, Annette	0	5	30	0:4	30:49

The four leftmost columns are piste allocations in rounds 1, 2, 3 and 4. ✓ = club-piste * = carpark-piste

BHN is the Buchholz Number that is the sum of the scores of the opponents.

fBHN is the Fine Buchholz Number that is the sum of the Buchholz Numbers of the opponents.

High Buchholz numbers indicate strong opponents (see teams ranked 1, 4, 5, 13, 14, 26, 27 and 28)

The rank order is games won (score), BHN, fBHN, points difference (see teams ranked 8 and 9; 20 and 21)

Six teams were allocated carpark-pistes in three of the qualifying rounds.

$$\Pr\{EEE\} = \Pr\{3 \text{ carpark-pistes}\} = \left(\frac{y}{n}\right)^3 = \left(\frac{8}{19}\right)^3 = 0.0746$$

14 teams were allocated carpark-pistes in two of the qualifying rounds.

$$\Pr\{EE\} = \Pr\{2 \text{ carpark-pistes}\} = \left(\frac{y}{n}\right)^2 = \left(\frac{8}{19}\right)^2 = 0.1773$$

14 teams were allocated a carpark-piste in one of the qualifying rounds.

$$\Pr\{E\} = \Pr\{1 \text{ carpark-pistes}\} = \frac{y}{n} = 0.4211$$

Three teams played all their qualifying rounds on club-pistes. The probability of this occurrence is.

$$\Pr\{4 \text{ club-pistes}\} = \left(\frac{x}{n}\right)^4 = \left(\frac{11}{19}\right)^4 = 0.1123$$

This would occur approximately 11 times in 100 similar tournaments.

In each qualifying round 16 teams were playing in 8 games in the carpark and 22 teams were playing in 11 games in the club. So for the four qualifying rounds a total of $4 \times 8 = 32$ games were played in the carpark involving 64 teams; obviously some teams played in the carpark on several occasions. Teams ranked somewhere from 1 to 16 played in the carpark on 28 occasions and teams ranked somewhere from 17 to 38 played in the carpark on 36 occasions so we might say:

Top 16 teams made up 28 of the 64 teams involved in carpark-games ($\frac{28}{64} = 43.75\%$)

Next 22 teams made up 36 of the 64 teams involved in carpark-games ($\frac{36}{64} = 56.25\%$)

In round 1, 6 of the Top 16 played in the carpark. In rounds 2, 3 and 4 it was 8, 9 and 5 respectively of the Top 16. Eight of the Top 16 played two or more games in the carpark.

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Monday, 09 March, 2015